Exercícios Cálculo - Área 3

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1. (Leithold, p. 148, exerc. 43-46) Ache $f'(a)$ usando a fórmula

$$f'(x_1) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

   (a) $f(x) = 4 - x^2$, $a = 5$
   (b) $f(x) = \frac{4}{5x}$, $a = 2$
   (c) $f(x) = \frac{2}{x^3}$, $a = 4$
   (d) $f(x) = \frac{2}{\sqrt{x}} - 1$, $a = 4$

2. (Leithold, p. 148, exerc. 47-50) Ache $f'(a)$ usando a fórmula

$$f'(x_1) = \lim_{x \to x_1} \frac{f(x) - f(x_1)}{x - x_1}$$

   (a) $f(x) = 2 - x^3$, $a = -2$
   (b) $f(x) = x^2 - x + 4$, $a = 4$
   (c) $f(x)\frac{1}{\sqrt{2x+3}}$, $a = 3$
   (d) $f(x) = \sqrt{1 + 9x}$, $a = 7$

3. (p. 149, exerc. 19-29) Encontre a derivada da função dada usando a definição

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

   Diga quais são os domínios da função e da derivada.

   (a) $f(x) = \frac{1}{2}x - \frac{1}{3}$
   (b) $f(x) = mx + b$
   (c) $f(t) = 5t - 9t^2$
   (d) $f(x) = 1,5x^2 - x + 3,7$
   (e) $f(x) = x^3 - 3x + 5$
   (f) $f(x) = x + \sqrt{x}$
   (g) $g(x) = \sqrt{1 + 2x}$
   (h) $f(x) = \frac{3+x}{1-3x}$

(i) $G(t) = \frac{4t}{t+1}$

(j) $g(x) = \frac{1}{\sqrt{x}}$

(k) $f(x) = x^4$

4. (Leithold, p. 162, exerc. 1-24) Calcule a derivada usando as propriedades:

   (a) $f(x) = 7x - 5$

   (b) $g(x) = 8 - 3x$

   (c) $g(x) = 1 - 2x - x^2$

   (d) $f(x) = 4x^2 + x + 1$

   (e) $f(x) = x^3 - 3x^2 + 5x - 2$

   (f) $f(x) = 3x^4 - 5x^2 + 1$

   (g) $f(x) = \frac{1}{8}x^8 - x^4$

   (h) $g(x) = x^7 - 2x^5 + 5x^3 - 7x$

   (i) $F(t) = \frac{1}{4}t^4 - \frac{1}{2}t^2$

   (j) $H(x) = \frac{1}{3}x^3 - x + 2$

   (k) $v(r) = \frac{4}{3}\pi r^3$

   (l) $G(y) = y^{10} + 7y^5 - y^3 + 1$

   (m) $F(x) = x^2 + 3x + \frac{1}{x^2}$

   (n) $f(x) = \frac{x^3}{3} + \frac{3}{x^3}$

   (o) $g(x) = 4x^4 - \frac{1}{4x^4}$

   (p) $f(x) = x^4 - 5 + x^{-2} + 4x^{-4}$

   (q) $g(x) = \frac{3}{x^2} + \frac{5}{x^4}$

   (r) $H(x) = \frac{5}{6x^5}$

   (s) $f(s) = \sqrt{3}(s^3 - s^2)$

   (t) $g(x) = (2x^2 + 5)(4x - 1)$

   (u) $f(x) = (2x^4 - 1)(5x^3 + 6x)$

   (v) $f(x) = (4x^2 + 3)^2$

   (w) $G(y) = (7 - 3y^3)^2$

   (x) $F(t) = (t^3 - 2t + 1)(2t^2 + 3t)$

5. (Leithold, p. 162-163, exerc. 25-36) Calcule a derivada usando as propriedades:

   (a) $D_x[(x^2 - 3x + 2)(2x^3 + 1)]$

   (b) $D_x\left(\frac{2x}{x+3}\right)$

   (c) $D_x\left(\frac{x}{x-1}\right)$

   (d) $D_y\left(\frac{2y+1}{3y+4}\right)$

   (e) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2+2x+1}{x^2-2x+1}\right)$

   (f) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4-3x-x^2}{x-2}\right)$

   (g) $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{5t}{1+2t^2}\right)$

   (h) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^4-2x^2+5x+1}{x^4}\right)$

   (i) $\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{y^3-8}{y^3+8}\right)$

   (j) $\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{s^2-a^2}{s^2+a^2}\right)$

   (k) $D_x\left[\frac{2x+1}{x+5}(3x-1)\right]$

   (l) $D_x\left[\frac{x^3+1}{x^2+3}(x^2 - 2x^{-1} + 1)\right]$

6. (p. 166, exerc. 6-24, 26-32) Derive a função:

(a) $F(x) = -4x^{10}$

(b) $f(x) = x^3 - 4x + 6$

(c) $f(t) = \frac{1}{2}t^6 - 3t^4 + t$

(d) $f(t) = \frac{1}{4}(t^4 + 8)$

(e) $h(x) = (x-2)(2x+3)$

(f) $y = x^{-2/5}$

(g) $y = 5e^x + 3$

(h) $V(r) = \frac{4}{3}\pi r^3$

(i) $R(t) = 5t^{-3/5}$

(j) $Y(t) = 6t^{-9}$

(k) $R(x) = \frac{\sqrt{10}}{x^7}$

(l) $G(x) = \sqrt{x} - 2e^x$

(m) $y = \sqrt[3]{x}$

(n) $F(x) = \left(\frac{1}{2}x\right)^5$

(o) $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$

(p) $y = ax^2 + bx + c$

(q) $y = \sqrt{x}(x-1)$

(r) $y = \frac{x^2+4x+3}{\sqrt{x}}$

(s) $y = \frac{x^2-2\sqrt{x}}{x}$

(t) $g(u) = \sqrt{2}u + \sqrt{3u}$

(u) $H(x) = (x + x^{-1})^3$

(v) $y = ae^v + \frac{b}{v} + \frac{c}{v^2}$

(w) $u = \sqrt[5]{t} + 4\sqrt{t^5}$

(x) $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$

(y) $z = \frac{A}{y^{10}} + Be^y$

(z) $y = e^{x+1} + 1$

7. (p. 166, exerc. 33 e 34) Encontre uma equação para a reta tangente no ponto dado:

(a) $y = \sqrt[4]{x}$, $(1, 1)$

(b) $y = x^4 + 2x^2 - x$, $(1, 2)$

8. (p. 166, exerc. 47 e 48) Encontre a primeira e a segunda derivadas da função. Verifique se suas respostas são razoáveis comparando os gráficos de f, f' e f''.

(a) $f(x) = 2x - 5x^{3/4}$

(b) $f(x) = e^x - x^3$

9. (p. 167, exerc. 67) Seja

$$f(x) = \begin{cases} 2 - x & \text{se } x \le 1 \\ x^2 - 2x + 2 & \text{se } x > 1 \end{cases}$$

f é derivável em 1? Esboce os gráficos de f e f'.

10. (p. 167, exerc. 68) Em quais números a seguinte função g é derivável?

$$g(x) = \begin{cases} -1-2x & \text{se } x < -1 \\ x^2 & \text{se } -1 \leq x \leq 1 \\ x & \text{se } x > 1 \end{cases}$$

Dê uma fórmula para g' e esboce os gráficos de g e g'.

11. (p. 172-173, exerc. 3-26) Derive:

(a) $f(x) = x^2e^x$

(b) $g(x) = \sqrt{x}e^x$

(c) $y = \frac{e^x}{x^2}$

(d) $y = \frac{e^x}{1+x}$

(e) $g(x) = \frac{3x-1}{2x+1}$

(f) $f(t) = \frac{2t}{4+t^2}$

(g) $V(x) = (2x^3+3)(x^4-2x)$

(h) $Y(u) = (u^{-2}+u^{-3})(u^5-2u^2)$

(i) $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y+5y^3)$

(j) $R(t) = (t+e^t)(3-\sqrt{t})$

(k) $y = \frac{x^3}{1-x^2}$

(l) $y = \frac{x+1}{x^3+x-2}$

(m) $y = \frac{t^2}{3t^2-2t+1}$

(n) $y = \frac{t^3+t}{t^4-2}$

(o) $y = (r^2-2r)e^r$

(p) $y = \frac{1}{s+ke^s}$

(q) $y = \frac{v^3-2v\sqrt{v}}{v}$

(r) $z = w^{3/2}(w+ce^w)$

(s) $f(t) = \frac{2t}{2+\sqrt{t}}$

(t) $g(t) = \frac{t-\sqrt{t}}{t^{1/3}}$

(u) $f(x) = \frac{A}{B+Ce^x}$

(v) $f(x) = \frac{1-xe^x}{x+e^x}$

(w) $f(x) = \frac{x}{x+\frac{c}{x}}$

(x) $f(x) = \frac{ax+b}{cx+d}$

12. (p. 180, exerc. 1-16) Derive:

(a) $f(x) = x - 3\sin x$

(b) $f(x) = x\sin x$

(c) $y = \sin x + 10\tan x$

(d) $y = 2\csc x + 5\cos x$

(e) $g(t) = t^3\cos t$

(f) $g(t) = 4\sec t + \tan t$

(g) $h(\theta) = \csc\theta + e^\theta\cot\theta$

(h) $y = e^u(\cos u + cu)$

(i) $y = \frac{x}{2-\tan x}$

(j) $y = \frac{1+\sin x}{x+\cos x}$

(k) $f(\theta) = \frac{\sec\theta}{1+\sec\theta}$

(l) $y = \frac{1-\sec x}{\tan x}$

(m) $y = \frac{\sin x}{x^2}$

(n) $y = \csc\theta(\theta+\cot\theta)$

(o) $f(x) = xe^x \csc x$

(p) $y = x^2 \sin x \tan x$

13. (p. 188, exerc. 7-21) Encontre a derivada da função.

(a) $F(x) = (x^3 + 4x)^7$

(b) $F(x) = (x^2 - x + 1)^3$

(c) $F(x) = \sqrt[4]{1 + 2x + x^3}$

(d) $f(x) = (1 + x^4)^{2/3}$

(e) $g(t) = \frac{1}{(t^4+1)^3}$

(f) $f(t) = \sqrt[3]{1 + \tan t}$

(g) $y = \cos(a^3 + x^3)$

(h) $y = a^3 + (\cos x)^3$

(i) $y = xe^{-kx}$

(j) $y = 3\cot(n\theta)$

(k) $g(x) = (1 + 4x)^5(3 + x - x^2)^8$

(l) $h(t) = (t^4 - 1)^3(t^3 + 1)^4$

(m) $y = (2x - 5)^4(8x^2 - 5)^{-3}$

(n) $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

(o) $y = \left(\frac{x^2+1}{x^2-1}\right)^3$

(p) $y = e^{-5x} \cos 3x$

(q) $y = e^{x \cos x}$

(r) $y = 10^{1-x^2}$

(s) $F(z) = \sqrt{\frac{z-1}{z+1}}$

(t) $G(y) = \frac{(y-1)^4}{(y^2+2y)^5}$

(u) $y = \frac{r}{\sqrt{r^2+1}}$

(v) $y = \frac{e^{2u}}{e^u+e^{-u}}$

(w) $y = \tan \cos x$

(x) $G(y) = \left(\frac{y^2}{y+1}\right)^5$

(y) $y = 2^{\sin \pi x}$

(z) $y = (\tan 3\theta)^2$

(aa) $y = (\sec x)^2 + (\tan x)^2$

(ab) $y = x \sin \frac{1}{x}$

(ac) $y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$

(ad) $F(t) = \sqrt{\frac{t}{t^2+4}}$

(ae) $y = (\cot(\sin\theta))^2$

(af) $y = e^{k \tan \sqrt{x}}$

(ag) $f(t) = \tan(e^t) + e^{\tan t}$

(ah) $y = \sin(\sin(\sin x))$

(ai) $f(t) = (\sin(e^{(\sin t)^2}))^2$

(aj) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(ak) $g(x) = (2ra^{rx} + n)^p$

(al) $y = 2^{3^{x^2}}$

(am) $y = \cos\sqrt{\sin(\tan \pi x)}$

(an) $y = (x + (x + (\sin x)^2)^3)^4$

14. (p. 197, exerc. 5,7,9,11,13,15,17 e 19) Encontre $\mathrm{d}y/\mathrm{d}x$ derivando implicitamente:

(a) $x^2 + y^3 = 1$

(b) $x^3 + x^2y + 4y^2 = 6$

(c) $x^4(x + y) = y^2(3x - y)$

(d) $x^2y^2 + x \sin y = 4$

(e) $4 \cos x \sin y = 1$

(f) $e^{x/y} = x - y$

(g) $\sqrt{xy} = 1 + x^2y$

(h) $xy = \cot(xy)$

15. (p. 204, exerc. 2, 3, 7, 11, 13, 15 e 19) Derive a função.

(a) $f(x) = \ln(x^2 + 10)$

(b) $f(x) = \sin(\ln x)$

(c) $f(x) = \sqrt[5]{\ln x}$

(d) $F(t) = \ln \frac{(2t+1)^3}{(3t-1)^4}$

(e) $g(x) = \ln(x\sqrt{x^2 - 1})$

(f) $y = \frac{\ln x}{1+x}$

(g) $y = \ln(e^{-x} + xe^{-x})$

16. (p. 267, exerc. 1-2) Verifique se a função satisfaz as três hipóteses do Teorema de Rolle no intervalo. Então, encontre todos os números c que satisfazem a conclusão do Teorema de Rolle.

(a) $f(x) = x^2 - 4x + 1$, $[0, 4]$

(b) $f(x) = x^3 - 3x^2 + 2x + 5$, $[0, 2]$

17. (p. 276, exerc. 9-12) Encontre os intervalos nos quais f é crescente ou decrescente e os valores máximos e mínimos locais de f.

(a) $f(x) = x^3 - 12x + 1$

(b) $f(x) = 5 - 3x^2 + x^3$

(c) $f(x) = x^4 - 2x^2 + 3$

(d) $f(x) = \frac{x^2}{x^2+3}$

18. (p. 284-285, exerc. 7, 9, 19 e 21) Encontre o limite. Use a Regra de L'Hôspital quando for apropriado.

(a) $\lim_{x\to 1} \frac{x^9-1}{x^5-1}$

(b) $\lim_{x\to(\pi/2)^+} \frac{\cos x}{1-\sin x}$

(c) $\lim_{x\to\infty} \frac{e^x}{x^3}$

(d) $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$